Integrēšanas METODES

I. Parciālā integrēšana:

$$\boxed{\int \mathbf{u}\cdot \mathbf{dv} = \mathbf{u}\cdot\mathbf{v} - \int \mathbf{v}\cdot \mathbf{du}}.$$

kur **a)** $\mathbf{u} = P_n(x)$, ja $\int P_n(x)\cdot f_{tiesā}\cdot dx$, kur $P_n(x)$ ir n-tās kārtas polinoms;

b) $\mathbf{u} = f_{inversā}$, ja $\int f_{inversā}\cdot f_{tiesā}\cdot dx$.

II. Trešā veida elementārdaļa:

$$\int \frac{\mathrm{M}\,\mathbf{x}+\mathrm{N}}{\mathbf{x^2}+\mathbf{px}+\mathbf{q}}\mathbf{dx} = \begin{bmatrix} (1):\ x^2+px+q = (x+a)^2+b, \quad a,b = const \\ (2):\ \text{subst. } [\mathbf{x+a=t}\,] \end{bmatrix}$$

III. Daļveida racionāla funkcija:

$$\int \frac{\mathbf{P_n(x)}}{\mathbf{Q_m(x)}}\mathbf{dx} = \begin{cases} \text{Ja } \mathbf{n \geq m}, \Rightarrow \text{neīsta daļa}, \Rightarrow \text{dala polinomus}, \\ \text{Ja } \mathbf{n < m}, \Rightarrow \text{īsta daļa}, \Rightarrow \text{shēma:} \end{cases}$$

(1) Sadala saucēju reizinātājos;

(2) Sadala daļu elementārdaļās ar nenoteiktiem koeficientiem:

$$\frac{1}{(x\pm a)\cdots} \to \frac{A}{x\pm a},$$

$$\frac{1}{(x^2+px+q)\cdots} \to \frac{Ax+B}{x^2+px+q},$$

$$\frac{1}{(x\pm a)^k\cdots} \to \frac{A}{x\pm a}+\frac{B}{(x\pm a)^2}+\frac{C}{(x\pm a)^3}+\cdots+\frac{W}{(x\pm a)^k},$$

(3) Aprēķina koeficientus.

IV. Trigonometriskās izteiksmes ($n,m = 1, 2, 3, \ldots$):

1.) tikai pāra pakāpes, t.i., $\int \sin^{2n}\mathbf{x}\cdot\cos^{2m}\mathbf{x}\,\mathbf{dx}$, $\int \sin^{2n}\mathbf{x}\,\mathbf{dx}$, $\int \cos^{2m}\mathbf{x}\,\mathbf{dx}$:

$$\begin{bmatrix} \cos^2 x = \frac{1}{2}(1+\cos 2x) \\ \sin^2 x = \frac{1}{2}(1-\cos 2x) \end{bmatrix} \quad \text{un} \quad \begin{bmatrix} \sin 2x = 2\sin x\cos x \\ \cos 2x = \cos^2 x - \sin^2 x \end{bmatrix}$$

2.) vismaz viena nepāra pakāpe, t.i.,

$\int \sin^{n}\mathbf{x}\cdot\cos^{2m+1}\mathbf{x}\,\mathbf{dx}$, $\int \sin^{2n+1}\mathbf{x}\cdot\cos^{m}\mathbf{x}\,\mathbf{dx}$, $\int \sin^{2n+1}\mathbf{x}\,\mathbf{dx}$, $\int \cos^{2m+1}\mathbf{x}\,\mathbf{dx}$:

$$\begin{bmatrix} \cos^{2m+1}x\,dx = \cos^{2m}x\cdot\cos x\,dx \\ \sin^{2n+1}x\,dx = \sin^{2n}x\cdot\sin x\,dx \end{bmatrix} \quad \text{un} \quad \begin{bmatrix} \cos^2 x = 1-\sin^2 x \\ \sin^2 x = 1-\cos^2 x \end{bmatrix}$$

3.) $\int \sin ax\cos bx\,dx$, $\int \sin ax\sin bx\,dx$, $\int \cos ax\cos bx\,dx$:

$$\begin{bmatrix} \cos ax\cdot\cos bx = \frac{1}{2}[\cos(a+b)x+\cos(a-b)x] \\ \sin ax\cdot\sin bx = \frac{1}{2}[\cos(a-b)x-\cos(a+b)x] \\ \sin ax\cdot\cos bx = \frac{1}{2}[\sin(a+b)x+\sin(a-b)x] \end{bmatrix}$$

4.) $\int \mathbf{R(\sin x,\cos x)\,dx}$ => **Universālā subst.** $\left[\mathbf{t = tg\frac{x}{2}}\right]$, => $x = 2\operatorname{arctg} t$.

$$\left[\sin x = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}\right]$$

5.) $\int \mathbf{R(tgx, ctgx, \{sinx, cosx\})\,dx}$

$\int \mathbf{R(\sin^{2n}x, \cos^{2m}x, sinx\cdot cosx)\,dx}$

$\Rightarrow$ Subst. $[\mathbf{t = tgx}]$, => $x = \operatorname{arctg} t$

$$\left[\sin x = \frac{\operatorname{tg} x}{\sqrt{1+\operatorname{tg}^2 x}} = \frac{t}{\sqrt{1+t^2}}, \quad \cos x = \frac{1}{\sqrt{1+\operatorname{tg}^2 x}} = \frac{1}{\sqrt{1+t^2}}\right] \quad \textbf{un} \quad \left[1+\operatorname{tg}^2 x = \frac{1}{\cos^2 x}, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x}, \quad \operatorname{tg} x = \frac{\sin x}{\cos x}, \quad \operatorname{ctg} x = \frac{\cos x}{\sin x}\right]$$

V. Iracionālas izteiksmes:

1.) $\int \mathbf{R\left(x, \sqrt[n]{ax+b}\right)dx}$ => Subst. $[\mathbf{ax+b=t^n}]$

2.) $\int \mathbf{R\left(x, \sqrt[n]{ax+b}, \sqrt[m]{ax+b}\right)dx}$ => $[\mathbf{ax+b=t^p}]$, kur p/n, p/m - veseli sk.

3.) $\int \mathbf{R\left(x, \sqrt{x^2-a^2}\right)dx}$ => Subst. $\left[\mathbf{x = \frac{a}{cost}}\right]$ un $\cos^2 x+\sin^2 x = 1$;

4.) $\int \mathbf{R\left(x, \sqrt{a^2-x^2}\right)dx}$ => Subst. $[\mathbf{x = a\,sint}]$ un $\cos^2 x+\sin^2 x = 1$;

5.) $\int \mathbf{R\left(x, \sqrt{x^2+a^2}\right)dx}$ => Subst. $[\mathbf{x = a\,tgt}]$ un $1+\operatorname{tg}^2 x = \frac{1}{\cos^2 x}$;

6.) $\int \mathbf{R\left(x, \sqrt{x^2+px+q}\right)dx}$ => pēc shēmas 3.veida elementārdaļām;

7.) $\int \frac{\mathbf{dx}}{\mathbf{x}\sqrt{\mathbf{x^2+px+q}}}$ => **Subst.** $\left[\mathbf{\frac{1}{x} = t}\right]$